Name

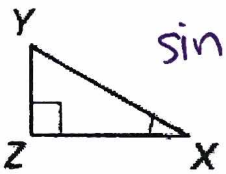
The Sine and Cosine Ratios

Lesson Objective: IWBAT apply the sine and cosine ratios in right Δs to find side lengths.

In the last lesson objective, we only saw one way to compare the three sides of a right triangle (called the Tangent Ratio). The two other combination of sides we will look at today are called the Sine and Cosine ratios.

The Sine Ratio

The sine ratio of an acute angle compares the leg opposite the acute angle and the hypotenuse



$\sin X = \frac{YZ}{YX}$

$\sin Y = \frac{XZ}{XY}$

$\sin \theta = \frac{\text{opp. leg}}{\text{hyp.}}$

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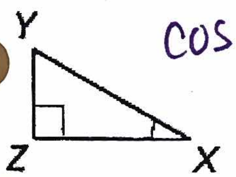
The Cosine Ratio

The cosine ratio of an acute angle compares the leg adjacent the acute angle and the hypotenuse



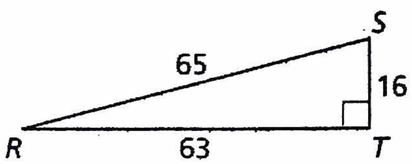
$\cos X = \frac{XZ}{YX}$

$\cos Y = \frac{YZ}{YX}$

$\cos \theta = \frac{\text{adj. leg}}{\text{hyp.}}$

Example:

1. Find $\sin S$, $\cos S$, $\sin R$ and $\cos R$. Leave each answer as a fraction simplified completely.



$\sin S = \frac{63}{65}$ $\sin R = \frac{16}{65}$

$\cos S = \frac{16}{65}$ $\cos R = \frac{63}{65}$

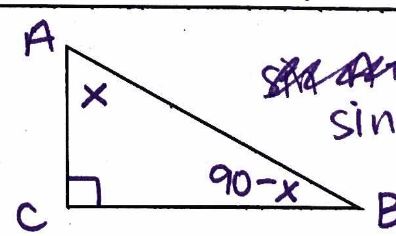
The sine **of an acute angle is equal to the** cosine **of its** complement.

Similarly, the cosine **of an acute angle is equal to the** sine **of its** complement.

Sine and Cosine of Complementary Angles

Let A and B be complementary angles in right ΔABC.

$\sin A = \cos B$ and $\cos A = \sin B$



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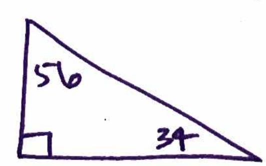
$\sin x = \cos(90-x)$

$\cos x = \sin(90-x)$

Example:

2. Write $\sin(56°)$ in terms of cosine.

$\cos(90-56) = \boxed{\cos(34°)}$



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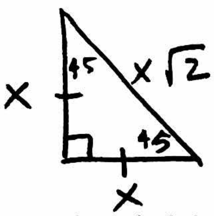
Sine and Cosine Ratios in Special Right Triangles

Examples:

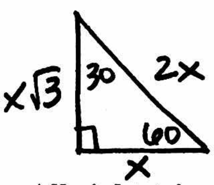
1. Find the sine and cosine of a 45° angle.



$\sin(45°) = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \boxed{\frac{\sqrt{2}}{2}}$

$\cos(45°) = \boxed{\frac{\sqrt{2}}{2}}$

2. Find the sine and cosine of a 30° angle.



$\sin(30°) = \frac{x}{2x} = \boxed{\frac{1}{2}}$

$\cos(30°) = \frac{x\sqrt{3}}{2x} = \boxed{\frac{\sqrt{3}}{2}}$

3. Find the sine and cosine of a 60° angle.

$\sin(60) = \frac{\sqrt{3}}{2}$

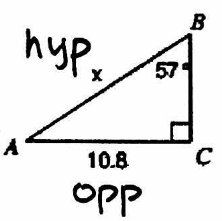
$\cos(60) = \frac{1}{2}$

All right triangles with a given acute angle are similar by AA~. So, regardless of the size of the triangle, the sine and cosine ratios will ALWAYS be the same for any given acute angle measure.

We can use this fact to find missing side lengths in right triangles.

Examples:

4. Find the value of x. Round your answer to the nearest hundredth.



$\sin(57) = \frac{10.8}{x}$

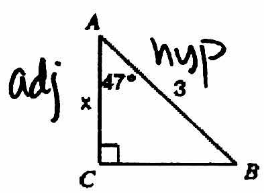
$\frac{0.8387}{1} = \frac{10.8}{x}$

$\rightarrow x(0.8387) = 10.8$

$x = \frac{10.8}{0.8387}$

$\boxed{x \approx 12.88}$

5. Find the value of x. Round your answer to the nearest hundredth.

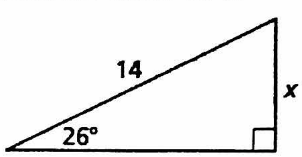


$\cos(47) = \frac{x}{3}$

$\frac{0.6820}{1} = \frac{x}{3}$

$\rightarrow \boxed{x \approx 2.05}$

6. Find the value of x.

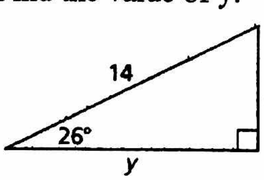


$\sin(26) = \frac{x}{14}$

$\frac{0.4384}{1} = \frac{x}{14}$

$\rightarrow \boxed{x \approx 6.14}$

7. Find the value of y.

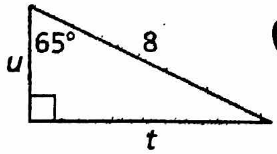


$\cos(26) = \frac{y}{14}$

$\frac{0.8988}{1} = \frac{y}{14}$

$\rightarrow \boxed{y \approx 12.58}$

8. Find the value of u. Then find the value of t.



$\cos(65) = \frac{u}{8}$

$0.4226 = \frac{u}{8} \rightarrow \boxed{u \approx 3.38}$

$\sin(65) = \frac{t}{8}$

$0.9063 = \frac{t}{8} \rightarrow \boxed{t \approx 7.25}$